Wave guide Theory and Diffraction of Electromagnetic waves :-

Electromagnetic waves propogating in space travel out in all directions. The power intensity of these waves decreases as the distance increases. The waveguide operates by confining the electromagnetic waves inside a metallic structure so that it does not spread out and losses resulting from this effect are eliminated. The wave guide refers to any linear structure that guides em waves between two end points. Typically it is a Transmission line comprising a hollow conducting tube, which may be rectangular or circular. The signal is confined by the total internal reflection from the walls of the wave guide.

Electromagnetic wave guides are analyzed by solving Maxwell's equations with boundary conditions determined by the properties of the material and their interfaces.
Wave guide propogation mode depends on the operating wavelength and polarization also on shape and size of the guide.

Putting in above eqn

$$n_d^{5/3} / n_{od}^{5/3} = \frac{n_{1d}^{5/3} + n_{od}^{5/3}}{(n_{od})^{5/3}}$$

or $$n_d^{5/3} = n_{od}^{5/3}\left[1 + \frac{n_{1d}}{n_{od}}\right]^{5/3}$$

Applying Binomial series and neglecting higher terms.

$$n_d^{5/3} = n_{od}^{5/3}\left(1 + \frac{5}{3}\frac{n_{1d}}{n_{od}}\right)$$

or $$n_d^{5/3} = n_{od}^{5/3}\left(\frac{5}{3}\frac{n_{1d}}{n_{od}}\right)$$

or $$n_d^{5/3} = \frac{5}{3} n_{1d}\, n_{od}^{5/3 - 1}$$

Putting the value of $n_d^{5/3}$ in eqn (a)

$$\left(\frac{n_d}{n_{od}}\right)^{5/3} = \frac{5/3\, n_{1d}\, n_{od}^{5/3-1}}{n_{od}^{5/3}}$$

$$\left(\frac{n_d}{n_{od}}\right)^{5/3} = \frac{5}{3} n_{1d} \frac{n_{od}^{2/3}}{n_{od}^{5/3}}$$

or $$\boxed{\left(\frac{n_d}{n_{od}}\right)^{5/3} = \frac{5}{3}\frac{n_{1d}}{n_{od}}}$$

Similarly Taking

$$\left(\frac{n_d}{n_{od}}\right)^{8/3} = \frac{n_d^{8/3}}{n_{od}^{8/3}} \quad \text{— (b)}$$

~~no~~ linearizing $n_d^{8/3}$

$$n_d^{8/3} = \left(n_{od} + n_{1d}\right)^{8/3}$$

$$n_d^{8/3} = n_{od}^{8/3}\left(1 + \frac{n_{1d}}{n_{od}}\right)^{8/3}$$

Applying Binomial series.

$$n_d^{8/3} = n_{od}^{8/3}\left(1 + \frac{8}{3}\frac{n_{1d}}{n_{od}}\right)$$

or $$n_d^{8/3} = n_{od}^{8/3}\left(\frac{8}{3}\frac{n_{1d}}{n_{od}}\right)$$

$$n_d^{8/3} = \frac{8}{3} n_{1d}\, n_{od}^{8/3-1}$$

Putting the value in eqs (b)

$$\left(\frac{n_d}{n_{od}}\right)^{8/3} = \frac{8/3\, n_{1d}\, n_{od}^{8/3-1}}{n_{od}^{8/3}}$$

or $$\boxed{\left(\frac{n_d}{n_{od}}\right)^{8/3} = \frac{8}{3}\frac{n_{1d}}{n_{od}}}$$

Putting the values of $\left(\frac{n_d}{n_{od}}\right)^{5/3}$ and $\left(\frac{n_d}{n_{od}}\right)^{8/3}$ in eqs (6)

$$\frac{dV_d}{dt} = \frac{-\nabla}{\rho_d}\left[\left(P_{oe}+P_{oi}\right)\frac{5}{3}\left(\frac{n_{1d}}{n_{od}}\right) + \frac{\alpha_1}{3}\left(T_{oe}^4 + T_{oi}^4\right)\frac{8}{3}\left(\frac{n_{1d}}{n_{od}}\right)\right] + \frac{1}{4\pi\rho_d}\left(\nabla\times B\right)\times B - \nabla\psi$$

Using vector identity

$$(\vec{\nabla} \times \vec{B}) \times \vec{B} = (\vec{B} \cdot \vec{\nabla}) \vec{B} - \vec{\nabla}(\vec{B} \cdot \vec{B})$$

linearizing $\vec{B}$ on R. H. S.

$$(\nabla \times B) \times B = (B_0 \cdot \nabla) B_1 - \nabla (B_0 \cdot B_1)$$

Putting the value in above eqs of motion and also linearized the equation

$$\frac{dV_{d1}}{dt} = -\frac{\nabla}{\rho_{0d}} \left[(P_{0e} + P_{0i}) \frac{5}{3} + \frac{\alpha_2}{3} (T_{0e}^4 + T_{0i}^4) \frac{8}{3} \right] \left(\frac{n_{1d}}{n_{0d}} \right)$$

$$+ \frac{1}{4\pi \rho_{0d}} \left[(B_0 \cdot \nabla) B_1 - \nabla (B_0 \cdot B_1) \right] - \nabla \psi_1$$

$$\frac{dV_{d1}}{dt} = -\frac{\nabla}{\rho_{0d}} \left[(P_{0e} + P_{0i}) \frac{5}{3} + \frac{\alpha_2}{3} (T_{0e}^4 + T_{0i}^4) \frac{8}{3} \right] \left(\frac{n_{1d}}{n_{0d}} \right)$$

$$+ \frac{1}{4\pi \rho_{0d}} (B_0 \cdot \nabla) B_1 - \frac{1}{4\pi \rho_{0d}} \nabla (B_0 \cdot B_1) - \nabla \psi_1$$

where V_{sd}^2 is dust acoustic velocity

$$V_{sd}^2 = \frac{5}{3} \left(\frac{P_{0e} + P_{0i}}{\rho_{0d}} \right)$$

basic equation

$$\therefore V = \sqrt{\frac{\gamma P}{\rho}}$$

And V_{rd}^2 is dust radiation velocity

$$V_{rd}^2 = \frac{8 \alpha_2}{9} \left(\frac{T_{0e}^4 + T_{0i}^4}{\rho_{0d}} \right)$$

where V_d^2 is the Jeans velocity

$$V_d^2 = V_{sd}^2 + V_{rd}^2$$

So equation of motion becomes

$$\boxed{\frac{dV_{d1}}{dt} = -\nabla V_d^2 \left(\frac{n_{1d}}{n_{0d}}\right) + \frac{1}{4\pi \rho_{0d}} (B_0 \cdot \nabla) B_1 + \frac{1}{4\pi \rho_{0d}} \nabla (B_0 \cdot B_1) - \nabla \psi_1}$$